Solution to Homework 7

Problem 1:

Consider the following simple Keynesian macroeconomic model of the U.S. economy.

$$Y_t = C_t + I_t + G_t + NX_t$$

$$C_t = \beta_0 + \beta_1 YD_t + \beta_2 C_{t-1} + \varepsilon_{1t}$$

$$YD_t = Y_t - T_t$$

$$I_t = \beta_3 + \beta_4 Y_t + \beta_5 r_{t-1} + \varepsilon_{2t}$$

$$r_t = \beta_6 + \beta_7 Y_t + \beta_8 M_t + \varepsilon_{3t}$$

where:

Y_t = gross domestic product (GDP) in year t
C_t = total personal consumption in year t
I_t = total gross private domestic investment in year t
G_t = government purchases of goods and services in year t
NX_t = net exports of goods and services (exports - imports) in year t
T_t = taxes in year t
r_t = the interest rate in year t
M_t = the money supply in year t
YD_t = disposable income in year t

The endogenous variables are Y_t, C_t, I_t, YD_t, and r_t. The exogenous and predetermined variables are Gt, NXt, Ct-1, Tt, rt-1, and Mt. Find the reduced form equations for this model.

Answer: Sub the second equation into the first:

$$Y_t = (\beta_0 + \beta_1 YD_t + \beta_2 C_{t-1} + \varepsilon_{1t}) + I_t + G_t + NX_t$$

Now sub for I_t and YD_t :

$$Y_t = (\beta_0 + \beta_1 (Y_t - T_t) + \beta_2 C_{t-1} + \varepsilon_{1t}) + (\beta_3 + \beta_4 Y_t + \beta_5 r_{t-1} + \varepsilon_{2t}) + G_t + NX_t$$

Now solve for Y_t :

$$Y_t = [1/(1-\beta_1-\beta_4)]\{(\beta_0 + \beta_3) - \beta_1 T_t + \beta_2 C_{t-1} + \beta_5 r_{t-1} + G_t + NX_t + \varepsilon_{1t} + \varepsilon_{2t}\}$$

And, in the form that would be estimated:

$$Y_t = \lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt}$$

Now substitute this solution into the other equations to find the rest of the reduced form relationships. Consumption first. Sub for disposable income, then income:

$$C_t = \beta_0 + \beta_1(Y_t - T_t) + \beta_2 C_{t-1} + \varepsilon_{1t}$$

$$C_t = \beta_0 + \beta_1[(\lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt}) - T_t] + \beta_2 C_{t-1} + \varepsilon_{1t}$$

$$C_t = (\beta_0+\beta_1\lambda_0) + \beta_1(\lambda_1-1)\, T_t + (\beta_1\lambda_2+\beta_2)C_{t-1} + \beta_1\lambda_3 r_{t-1} + \beta_1\lambda_4 G_t + \beta_1\lambda_5 NX_t + (\beta_1\varepsilon_{yt} + \varepsilon_{1t})$$

Next, YD_t:

$$YD_t = Y_t - T_t = Y_t = \lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt} - T_t$$

$$YD_t = = \lambda_0 + (\lambda_1-1)T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt}$$

Investment is next on the list:

$$I_t = \beta_3 + \beta_4 Y_t + \beta_5 r_{t-1} + \varepsilon_{2t}$$

$$I_t = \beta_3 + \beta_4(\lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt}) + \beta_5 r_{t-1} + \varepsilon_{2t}$$

$$I_t = (\beta_3 + \beta_4\lambda_0) + \beta_4\lambda_1 T_t + \beta_4\lambda_2 C_{t-1} + (\beta_4\lambda_3 + \beta_5)r_{t-1} + \beta_4\lambda_4 G_t + \beta_4\lambda_5 NX_t + (\beta_4\varepsilon_{yt} + \varepsilon_{2t})$$

And finally, the interest rate:

$$r_t = (\beta_6 + \beta_7\lambda_0) + \beta_7\lambda_1 T_t + \beta_7\lambda_2 C_{t-1} + \beta_7\lambda_3 r_{t-1} + \beta_7\lambda_4 G_t + \beta_7\lambda_5 NX_t + \beta_8 M_t + (\beta_7\varepsilon_{yt} + \varepsilon_{3t})$$

Problem 2:

9.3 A researcher is investigating the impact of advertising on sales using cross-sectional data from firms producing recreational goods. For each firm there are data on sales, S, and expenditure on advertising, A, both measured in suitable units, for a recent year. The researcher proposes the following model:

$$S = \beta_1 + \beta_2 A + u_S$$

$$A = \alpha_1 + \alpha_2 S + u_A$$

where u_S and u_A are disturbance terms. The first relationship reflects the positive effect of advertising on sales, and the second the fact that largest firms, as measured by sales, tend to spend most on advertising. Give a mathematical analysis of what would happen if the researcher tried to fit the model using OLS.

Answer: First we will derive the reduced from equation for A. Substituting for S in the second equation,

$$A = \alpha_1 + \alpha_2(\beta_1 + \beta_2 A + u_S) + u_A$$

Hence

$$A = \frac{\alpha_1 + \alpha_2\beta_1 + \alpha_2 u_S + u_A}{1 - \alpha_2\beta_2}$$

Thus the regression model assumption that A is distributed independently of u_S is violated and OLS will yield inconsistent estimates if used to fit the first equation:

$$b_2^{\text{OLS}} = \frac{\sum (A_i - \bar{A})(S_i - \bar{S})}{\sum (A_i - \bar{A})^2} = \beta_2 + \frac{\sum (A_i - \bar{A})(u_{Si} - \bar{u}_S)}{\sum (A_i - \bar{A})^2}$$

Hence

$$\operatorname{plim} b_2^{\text{OLS}} = \beta_2 + \frac{\operatorname{plim}\frac{1}{n}\sum (A_i - \bar{A})(u_{Si} - \bar{u}_S)}{\operatorname{plim}\frac{1}{n}\sum (A_i - \bar{A})^2} = \beta_2 + \frac{\operatorname{cov}(A, u_S)}{\operatorname{var}(A)}$$

$$= \beta_2 + \frac{\operatorname{cov}\left(\left[\frac{\alpha_1 + \alpha_2\beta_1 + \alpha_2 u_S + u_A}{1 - \alpha_2\beta_2}\right], u_S\right)}{\operatorname{var}\left(\frac{\alpha_1 + \alpha_2\beta_1 + \alpha_2 u_S + u_A}{1 - \alpha_2\beta_2}\right)}$$

$$= \beta_2 + (1 - \alpha_2\beta_2)\frac{\alpha_2\sigma_{u_S}^2}{\alpha_2^2\sigma_{u_S}^2 + \sigma_{u_A}^2}$$

assuming that u_S and u_A are distributed independently of each other. Stability requires $\alpha_2\beta_2 < 1$, and it is reasonable to suppose that α_2 is positive, so the bias is upwards. Since the model is symmetrical in S and A, the OLS estimator of the slope coefficient in the second equation would also be upwards biased.