Economics 421
Winter 2008
Midterm 1

1. Consequences of omitting a relevant variable are:

(a) If omitted variable is correlated with one of the X^s → bias and inconsistency

(b) Even if uncorrelated with X^s, constant estimate is biased [but, if no corr, other $\hat{\beta}^s$ are unbiased]

(c) σ_u^2 estimated incorrectly

(d) So, since σ_u^2 wrong, confidence intervals are incorrect (so are test stats).

(e) When biased β^s, forecasts are unreliable

[cont.]

① [cont.]

Consequences of Including an irrelevant variable:

(a) OLS is unbiased, consistent

(b) variance correctly estimated

(c) Test stats valid

(d) But, inefficient

Overall, though, the consequences in this case, particularly the lack of bias, are smaller.

It comes down to bias versus inefficiency, and bias is usually the more severe problem.

(2) (a) t-stat is $\frac{-1.64}{.41}$

(b) Critical t is 2.028

(c) I mean larger in magnitude, i.e. that $\left|\frac{-1.64}{.41}\right| > 2.028$

(which it is). If so, then you would reject the null that $\beta_4 = 0$ in favor of $\beta_4 \neq 0$, the alternative

(d) $$F = \frac{\frac{SSR_R - SSR_u}{\# \text{ Rest}}}{\frac{SSR_u}{N-K}} = \frac{\frac{246-221}{2}}{\frac{221}{40-4}}$$

Critical F is $F_{.05}(2, 36) = 3.27$

(that's for N=35 which is closest value

(3) The violation is of the assumption that all the error terms have identical variances, i.e. that $E(u_i^2) = \sigma^2$ for all i.

The consequences are that

(a) OLS is still unbiased and consistent

(b) estimates of $\hat{\beta}^S$ are not efficient, $Var(\hat{\beta})$ is biased, inconsistent

(c) since $\hat{V}(\hat{\beta})$ is biased and inconsistent, test stats are wrong

(d) Forecasts are unbiased, but not efficient.

(4) For part (a), the steps are

(S1) Regress Y_i on a constant, X_2, and X_3, and save the residuals in a series called "resid"

(S2) Form a new variable resid2 = resid * resid, ie, the square of the residual

(S3) Regress resid2 on a constant, $X2$, and $X3$, and form the χ^2 statistic

TR^2

$\hookrightarrow$ # of obs

IF TR^2 calculated $>$ χ^2 (2 d.f.) from table, reject homoskedasticity, else fail to reject

(4) [cont.]

For part (b), the correction, do the following:

(S1) go back to the regression of resid2 [in step (S3) of Part (a)] on C, X2, and X3, or repeat (S1) through (S3).

(S2) From the regression of $\underbrace{\hat{u}_i^2}_{= \text{Resid2}}$ on C, X2, and X3, calculated the forecasted values, call them resid2f. Generate a new variable, resid f = $\sqrt{\text{resid2F}}$

(S3) divide Y, C, X2, and X3 by residf, then regress

$$\frac{Y}{\text{residf}} \text{ on } \frac{C}{\text{residf}}, \frac{X2}{\text{residf}}, \text{ and } \frac{X3}{\text{residf}}.$$

(S4) Go home. You are done.

⑤ The assumption that u_t is uncorrelated with u_s $(t \neq s)$ is violated.

The consequences are:

ⓐ OLS remains unbiased and consistent

ⓑ But, OLS not BLUE (inefficient)

ⓒ If X's include lagged dep. var, e.g.

$$Y_t = \alpha + \beta_1 Y_{t-1} + \beta_2 X_t + u_t$$

and u's are serially corr

$$u_t = \rho u_{t-1} + e_t,$$

then biased and inconsistent estimators of β's.

[cont.]

(5) [cont.)

(d) If pos serial correlation ($\rho > 0$ but < 1), and X^s trend upward,

$\hat{\sigma}_u^2$ too small

R^2 too large

(e) generally $\hat{\sigma}_{\hat{\beta}}$ biased

($\hat{\sigma}_u^2$ too small

$\rightarrow$ $\hat{\sigma}_{\hat{\beta}}^2$ too small

$\rightarrow$ t too big $\rightarrow$ too many rejections

If pos corr and X^s trend↑ as in (d)

(f) Generally, t, F

biased, inconsistent,

hence, invalid.

(6) (a) $\hat{\beta}_2 = \dfrac{\sum(X_i - \bar{X})(y_i - \bar{y})}{\sum (X_i - \bar{x})^2}$. Sub in

that $y_i - \bar{y} = \beta(X_i - \bar{x}) + u_i$ to get

$$\hat{\beta}_2 = \frac{\sum(X_i - \bar{x})[\beta(X_i - \bar{x}) + u_i]}{\sum (X_i - \bar{x})^2}$$

$$= \beta \frac{\sum (X_i - \bar{x})^2}{\sum (X_i - \bar{x})^2} + \frac{\sum (X_i - \bar{x}) u_i}{\sum (X_i - \bar{x})^2}$$

$$\hat{\beta} = \beta + \frac{\sum (X_i - \bar{x}) u_i}{\sum (X_i - \bar{x})^2}$$, let $a_i = \dfrac{(X_i - \bar{x})}{\sum (X_i - \bar{x})^2}$

Then $E(\hat{\beta}) = \beta + E(\sum a_i u_i)$

$= \beta + \sum E(a_i u_i)$

<u>IF</u> a_i, u_i are independent, then

$E(a_i u_i) = E(a_i) E(u_i) = 0$

since $E(u_i) = 0$

(b) a_i is a function of <u>all</u> the x^s, so u_i must be independent of all x^s, not just x_i

[cont]

(6) [cont.]

Consider a model with a lagged Y:

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 Y_{t-1} + u_t$$

forward one period

These are correlated

$$Y_{t+1} = \beta_1 + \beta_2 X_{2,t+1} + \beta_3 Y_t + u_{t+1}$$

Since u_t correlated with Y_t, "one of the X^s", we have bias

(c) bias measures the expected outcome in repeated samples for a given sample size.

Consistency looks at what happens to the estimator as $T \to \infty$ (or $N \to \infty$ depending on notation).

(7) We want to test that $\beta_2 - 2\beta_3 = 1$ using the model $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$

steps

(1) Estimate the UR model with OLS: $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$

Save the RSS_{UR} from the output

Also note that $k = 3$.

(2) Impose the restriction and estimate the UR model. Restriction is $\beta_2 = 2\beta_3 + 1$

$$Y_i = \beta_1 + (2\beta_3 + 1) X_{i2} + \beta_3 X_{i3} + u_i$$

group terms

$$Y_i - X_{i2} = \beta_1 + \beta_3 (2X_{i2} + X_{i3}) + u_i$$

So, Regress $Y_i - X_{i2}$ [form this using genr command, in Excel, etc.] on a constant, and on $(2X_{i2} + X_{i3})$, then save RSS_R

[cont.]

⑦ [continued]

③ Now form

$$F = \frac{\frac{RSS_R - RSS_{UR}}{1}}{\frac{RSS_{UR}}{N-3}}$$

and

Compare to .05 critical $F(1, N-K)$ from the table.

If $F_{critical} > F_{calculated}$ reject the restriction, else fail to reject.