Economics 421

Winter 2008

Midterm 2

(1) $$y_t^* = \beta_0 + \beta_1 X_t^* + V_t$$

$$y_t - W_t = \beta_0 + \beta_1 (X_t - W_t) + V_t$$

$$y_t = \beta_1 + \beta_1 X_t + \underbrace{V_t + (1-\beta_1) W_t}_{u_t}$$

This will be inconsistent. To see this,

$$\text{Plim}\, \hat{\beta}_1^{OLS} = \beta_1 + \frac{\text{Cov}(X, u)}{\text{Var}(X)} = \beta_1 + \frac{\text{Cov}(X^* + W, V_t + (1-\beta_1) W)}{\text{Var}(X)}$$

$$\text{Plim}\, \hat{\beta}_1^{OLS} = \beta_1 + \frac{(1-\beta_1)\sigma_w^2}{\sigma_{x^*}^2 + \sigma_w^2} \Rightarrow \text{inconsistent}$$

(2) (a) $d_L = 1.55$ $\quad d = 1.62$

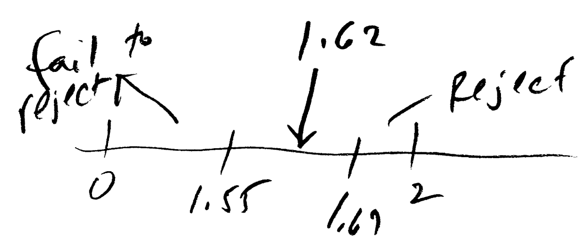
$K = 3$ $\quad d_u = 1.67$

$N = 70$

$\Rightarrow$ test is inconclusive

$H_0: \rho = 0$

$H_1: \rho > 0$



[cont.]

(2) (b) Suppose the model is

$$y_t = \beta_1 + \beta_2 X_{2t} + \cdots + \beta_K X_{Kt} + u_t$$

$$u_t = \rho_1 u_{t-1} + \cdots + \rho_p u_{t-p} + \varepsilon_t$$

steps

(1) Estimate $y_t = \beta_1 + \beta_2 X_{2t} + \cdots + \beta_K X_{Kt} + u_t$ by OLS, save residuals $(\hat{u}_t)$

(2) Regress $\hat{u}_t$ on $X_{2t} \cdots X_{Kt}$
$\hat{u}_{t-1} \cdots \hat{u}_{t-p}$

# of observations is $N-p$ since there are p lags

(3) Compute $(N-p)R^2$

If $(N-p)R^2 > \chi^2_p(a)$ — a: level of significance, e.g. 5% — p: degrees of freedom

reject null that all the ρs are zero

$H_0: \rho_1 = \rho_2 = \cdots = \rho_p = 0$

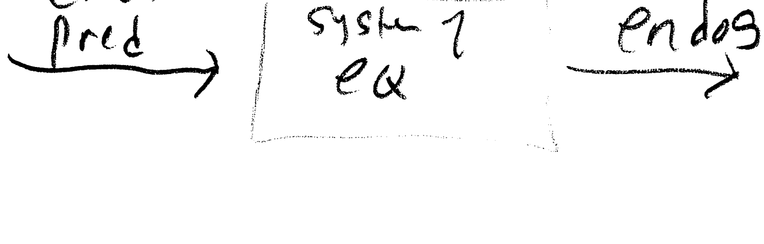
H_1: At least one is non-zero

(3) (a) (i) An inconsistent estimator does <u>not</u> coverge to the true value as $N \to \infty$, i.e. $\text{plim } \hat{\beta} \neq \beta$.

(ii) A reduced form expresses all of the endogenous variables as function of (just the) exogenous and predetermined variables, i.e. endog = f(exog, predet).

(iii) exogenous or predetermined variables are determined outside the model. They are the data that is input into the model to determine the endogenous variables

(iv) endogenous variables are determined within the system of eQ$^{\underline{s}}$



[Cont.]

(3) (b) Durbin's h-test is used when there are lagged endogenous variables, e.g.

$$y_t = c + \rho y_{t-1} + \beta x_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

Because y_{t-1} is correlated with the error term (since y_{t-1} is corr with u_{t-1}, and u_{t-1} is in u_t), the estimate of $\hat{u}_t$ is biased/inconsistent → biased DW stat (since it depends upon $\hat{u}_t$). Durbin's h-stat solves this problem

(4) (a) A valid IV is

Correlated w/ the variable it is instrumenting for

Uncorrelated with the error term

Not one of the other $X^{\underline{s}}$ already in the equation

[cont]

(4) (b) Suppose that

$$y_t = \beta_1 + \beta_2 x_t + u_t$$

where $\operatorname{cov}(x_t, u_t) \neq 0$

$\Rightarrow$ biased.

Let z_t be IV for x_t, i.e

$\operatorname{cov}(x\, z) \neq 0$

$\operatorname{cov}(z, u) = 0$

Then $$\hat{\beta}^{IV} = \frac{\sum (z_i - \bar{z})(y_i - \bar{y})}{\sum (z_i - \bar{z})(x_i - \bar{x})}$$

$$= \frac{\sum (z_i - \bar{z})[\beta_2 (x_i - \bar{x}) + (u_i - \bar{u})]}{\sum (z_i - \bar{z})(x_i - \bar{x})}$$

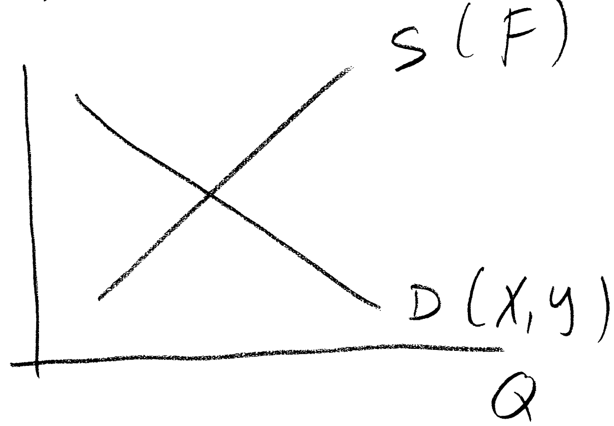
$$= \frac{\beta_2 \sum (z_i - \bar{z})(x_i - \bar{x})}{\sum (z_i - \bar{z})(x_i - \bar{x})} + \frac{\sum (z_i - \bar{z})(u_i - \bar{u})}{\sum (z_i - \bar{z})(x_i - \bar{x})}$$

$$\hat{\beta}_2^{IV} = \beta_2 + \frac{\frac{1}{N} \sum (z_i - \bar{z})(u_i - \bar{u})}{\frac{1}{N} \sum (z_i - \bar{z})(x_i - \bar{x})}$$ Take plim

$$\operatorname{plim} \hat{\beta}_2^{IV} = \beta_2 + \frac{\operatorname{cov}(z, u)}{\operatorname{cov}(z, x)} = \beta_2 + \frac{0}{\operatorname{cov}(z, x)} = \beta_2$$

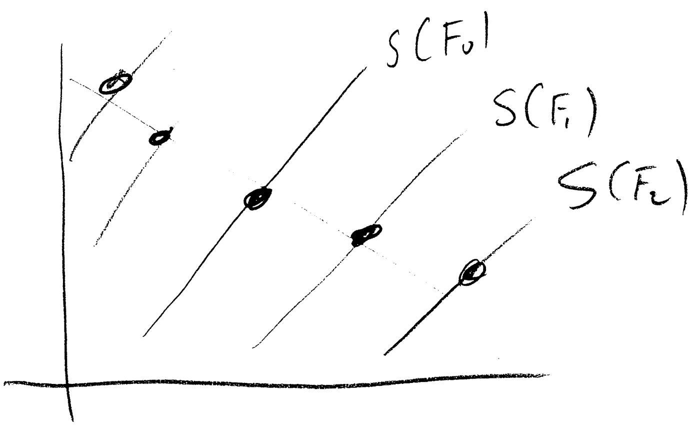
So, consistent.

⑤ The model can be represented graphically as:



<u>Demand Equation:</u> This is identified by movements in the supply curve.

These will map out the D-curve, though not perfectly due to randomness.



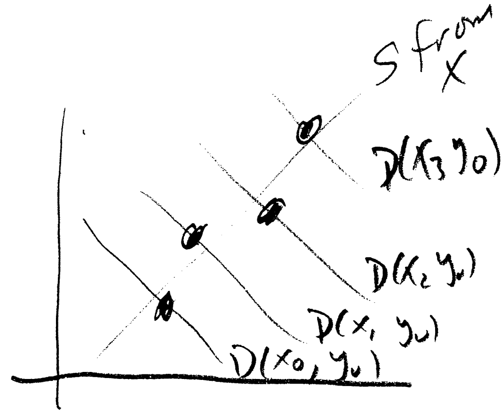
But fitting a line through the "dots" will give an estimate of the d-curve

In this case, there is only one set of points so exactly ID

[contrast with next case]

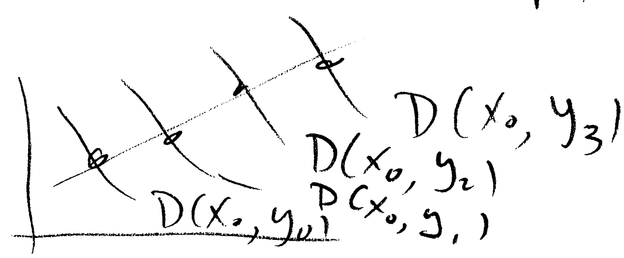
Supply equation

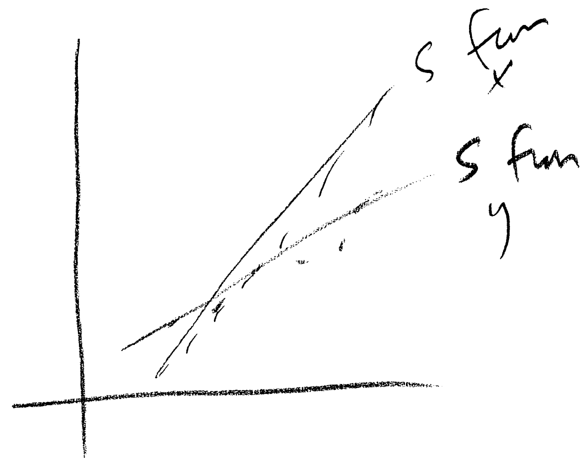
here, movements in X → one S-equation



(y stays at y_0, X "maps out" supply

movements in y → Another S-equation





Put together, have two estimates (so, over ID). To resolve, choose line closest to the two estimates

(6) (a) Two versions (some people did problem before I corrected the typo, $y = C+I+G$ should be $y = C+I+G+NX$).

Original (as written, only a few did this way, it should be noted).

$$y = C + I + G$$
$$C = a + b(y-T) + u$$
$$NX = f + gY + hP + v$$

$$y = a + b(y-T) + u + I + G$$

$$\rightarrow \quad y = \frac{1}{1-b}\left[a - bT + I + G + u\right]$$

$$C = a + b[Y - T] + u$$

$$C = a + b\left[\frac{1}{1-b}(a - bT + I + G + u) - T\right] + u$$

$$C = \left(a + \frac{ba}{1-b}\right) - \frac{b^2}{1-b}T - bT + \frac{b}{1-b}(I+G+u) + u$$

$$\rightarrow \quad C = \frac{a}{1-b} - \frac{b}{1-b}T + \frac{b}{1-b}(I+G) + \frac{1}{1-b}u$$

okay to leave like this

$$NX = f + \frac{g}{1-b}\left\{a - bT + I + G + u\right] + hP + v$$

$$\rightarrow \quad NX = \left(f + \frac{ga}{1-b}\right) - \frac{bg}{1-b}T + \frac{g}{1-b}(I+G+u) + hP + v$$

{cont.}

6 (a) Now, correct typo.

$$Y = C + I + G + NX$$
$$C = a + b(Y - T) + u$$
$$NX = f + gY + hP + v$$

$$Y = a + b(Y-T) + u + I + G + f + gY + hP + v$$

$$Y - bY - gY = a + f - bT + I + G + hP + u + v$$

$$\rightarrow Y = \frac{1}{1-b-g}\left[a + f - bT + I + G + hP + \underbrace{u + v}_{\varepsilon}\right]$$

write as

$$Y = \lambda_0 + \lambda_1 T + \lambda_2 I + \lambda_3 G + \lambda_4 P + \varepsilon$$

$$C = a + b(\lambda_0 + \lambda_1 T + \lambda_2 I + \lambda_3 G + \lambda_4 P + \varepsilon) - bT + u$$

$$\rightarrow C = (a + b\lambda_0) + (b\lambda_1 - b)T + b\lambda_2 I + b\lambda_3 G + b\lambda_4 P + b\varepsilon + u$$

$$NX = f + g(\lambda_0 + \lambda_1 T + \lambda_2 I + \lambda_3 G + \lambda_4 P + \varepsilon) + v$$

$$\rightarrow NX = (f + g\lambda_0) + g\lambda_1 T + g\lambda_2 I + g\lambda_3 G + g\lambda_4 P + g\varepsilon + v$$

6(b) $Q_t = a_0 + a_1 P + a_2 Y + a_3 X + a_4 Z + u \rightarrow$ # exc $= 1 < G-1$ (Not ID)

$G-1 = 2$

$P = b_0 + b_1 Q + b_2 Y + b_3 W + v \rightarrow$ # exclud $= 2 = G-1$ (exact ID)

$Y = C_0 + C_1 P + C_2 W + w \rightarrow$ # exclud $= 3 > G-1$ (over ID)

First: under
Second: exact
Third: over

(7) Find Reduced form

$$S = \beta_1 + \beta_2 A + u_S$$

$$A = \alpha_1 + \alpha_2 S + u_A$$

$$= \alpha_1 + \alpha_2(\beta_1 + \beta_2 A + u_S) + u_A$$

$$A = \frac{1}{1-\alpha_2\beta_2}\left\{\alpha_1 + \alpha_2\beta_1 + \alpha_2 u_S + u_A\right\}$$

{don't need S equation}

Then $$\hat{\beta}_2^{OLS} = \frac{\Sigma(S_i-\bar{S})(A_i-\bar{A})}{\Sigma(A_i-\bar{A})^2} = \beta_2 + \frac{\frac{1}{N}\Sigma(A_i-\bar{A})(u_{S_i}-\bar{u}_S)}{\frac{1}{N}\Sigma(A_i-\bar{A})^2}$$

$$\text{Plim}\,\hat{\beta}_2^{OLS} = \beta_2 \; \frac{\text{Cov}(A, u_S)}{\text{Var}(A)}$$

$$= \beta_2 + \frac{\text{Cov}\left[\frac{\alpha_1+\alpha_2\beta_1+\alpha_2 u_S + u_A}{1-\alpha_2\beta_2}, u_S\right]}{\text{VAR}\left\{\frac{\alpha_1+\alpha_2\beta_1+\alpha_2 u_S+u_A}{1-\alpha_2\beta_2}\right\}}$$

$$= \beta_2 + (1-\alpha_2\beta_2)\frac{\alpha_2\sigma_{u_S}^2}{\alpha_2^2\sigma_{u_S}^2 + \sigma_{u_A}^2}$$