Sketch of a Model of Microsoft's Social Value

Robert Barro, June 2007

Goods are produced by competitive firms using the freely accessible production function:

$$(1) \qquad Y = AL^{1-\alpha} \cdot \left[\sum_{j=1}^{N} (x_j)^{\sigma} \right]^{\alpha/\sigma} \cdot N^{(\sigma-\alpha)/\sigma},$$

where A>0, L is labor input, x_j is the quantity of intermediate input of type j, and N is the number of varieties of intermediates that exist. The quantity L is in fixed aggregate supply. Although L is called labor, it really represents all of the usual rival inputs to production (unskilled labor, skilled labor, capital—all treated here as in fixed aggregate supply). Software and other idea-type goods are modeled as the intermediates. These goods are treated, for simplicity, as non-durables. The parameter α (0<α<1) will be the income share for intermediates. The parameter σ (0<σ<1) measures substitutability among types of intermediates. The presence of the last term in Eq. (1) will imply that total gross output, Y, is proportional to N, and this property will allow for endogenous growth in dynamic models where N grows due to R&D activity. The present analysis considers only one-time shifts in N.

Suppose that an intermediate of type j is priced at P_j>0. Competitive, profit-maximizing producers of final output equate the marginal product of x_j to P_j. This condition yields the demand function:

$$(2) \qquad x_j = L^{(1-\alpha)/(1-\sigma)} \cdot \left\{ A\alpha N^{(\sigma-\alpha)/\sigma} \cdot \left[\sum_{j=1}^{N} (x_j)^{\sigma} \right]^{(\alpha-\sigma)/\sigma} \right\}^{1/(1-\sigma)} \cdot (P_j)^{-1/(1-\sigma)} .$$

Hence, if N is large, the elasticity of demand for x_j is approximately constant and equal to $-1/(1-\sigma)$, which exceeds one in magnitude. (Competitive producers of final goods hire labor at a given wage rate, w. In equilibrium, w equals the marginal product of labor, and each producer of final goods earns zero profit.)

Each type of intermediate, x_j, is produced at constant marginal (and average) cost, $c>0$. Without loss of generality, assume $c=1$. Thus, physically, a unit of x_j is "produced" by taking a unit of final output and placing a j-type label on it. This labeling is assumed to be the exclusive province of intermediate firm j, which owns the rights to produce that intermediate. (This exclusive holder may be the inventor or developer.) The perpetual profit flow for intermediate firm j is

$$\pi_j = (P_j - 1)\cdot x_j \; . \tag{3}$$

Intermediate firm j chooses P_j (at each point in time) to maximize π_j, subject to Eq. (2). This condition yields the monopoly price, $(P_j)^*$:

$$(P_j)^* = 1/\sigma. \tag{4}$$

Hence, the monopoly price is the markup, $1/\sigma$, of marginal cost, 1.

We can generalize from pure monopoly to assume that each firm j actually prices as the fraction λ of the monopoly price:

$$P_j = \lambda/\sigma, \tag{5}$$

where $\sigma \leq \lambda \leq 1$. The first part of the inequality ensures that profit is non-negative. The monopoly case corresponds to $\lambda=1$.

Since the model is fully symmetric across types of intermediates, the values of P_j, x_j, and π_j are the same for all j. Denote these values by P, x, and π. We can use the results for x to determine total output (gross of production of intermediates) from Eq. (1) to be

(6) $$Y = A^{1/(1-\alpha)} \cdot (\frac{\alpha\sigma}{\lambda})^{\alpha/(1-\alpha)} LN .$$

Total output goes to aggregate consumption, C, and aggregate intermediate production, Nx. (This model excludes investment, including R&D outlays that might lead to changes in N over time.) Total profit is $N\pi$. Consumption is divided among wage earners and owners of intermediate firms. The part of consumption that goes to the wage earners is $C- N\pi$.

We can readily work out formulas for all of these variables. It is convenient to express the results as ratios to Y, given by Eq. (6). The various ratios turn out to be:

(7) $$Nx/Y = \alpha\sigma/\lambda,$$

(8) $$NPx/Y = \alpha,$$

(9) $$C/Y = 1 - \alpha\sigma/\lambda,$$

(10) $$N\pi/Y = \alpha\cdot(\lambda-\sigma)/\lambda,$$

(11) $$(C-N\pi)/Y = 1-\alpha.$$

The variable NPx is the total revenue of intermediate firms. The ratio of wage-earner consumption to this revenue follows from Eqs. (11) and (8) as

(12) $$(C-N\pi)/NPx = (1-\alpha)/\alpha.$$

Note that the last ratio depends only on α (the share of intermediate factor income in total income) and not on σ (substitutability among intermediates) or λ (markup ratio relative to the monopoly markup).

If N increases, Y rises in accordance with Eq. (6). The other variables (Nx, NPx, C, $N\pi$, $C-N\pi$) rise in the same proportion—that is, the ratios given in Eqs. (7)-(11) are constants. We can think of the creation of Microsoft as raising N (adding a variety of intermediate product, corresponding to Windows and other software). We can think of Microsoft's observed gross revenue (say $44 billion per year) as the addition to NPx. Therefore, Eq. (12) implies that the addition to wage-earner consumption (that is, consumption beyond that enjoyed by owners of Microsoft) is $44 billion multiplied by $(1-\alpha)/\alpha$.

The parameter α represents the share of total income going to intermediate production—that is, inputs that have an idea-type character. It seems that much of national income would flow to standard, rival-type factors of production, so that α would be well below one-half. Hence, $(1-\alpha)/\alpha$ tends to be well above one. My "conservative" calculation assumed that $(1-\alpha)/\alpha$ equaled one.

This calculation gives no weight to the added consumption of Microsoft owners (including Bill Gates). This additional consumption corresponds to the rise in $N\pi$. The additional term follows from Eqs. (10) and (8) as $1 - \sigma/\lambda$ (which has to be non-negative). That is, this term adds to $(1-\alpha)/\alpha$ to incorporate the added consumption of Microsoft owners. (Note that this analysis treats the increase in N as coming without cost. In a dynamic analysis, changes in N could be related to costly R&D outlays.)