1. Using data on X instead of Z in an OLS regression gives the following estimate for β_2:

$$\hat{\beta}_2 = \beta_2 + \frac{\sum(x_i - \bar{x})[(v_i - \beta_2 w_i) - (\bar{v} - \beta_2 \bar{w})]}{\sum(x_i - \bar{x})^2}$$

Then:

$$plim\ \hat{\beta}_2 = \beta_2 + \frac{cov(x, v - \beta_2 w)}{var(x)}$$

But since $x = z + w$, x and w are correlated with each other, so the estimator of β_2 is inconsistent. In particular:

$$plim\ \hat{\beta}_2 = \beta_2 - \beta_2 \frac{\sigma_w^2}{\sigma_x^2}$$

The estimator for β_1 is:

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2 \bar{x}$$

Because the estimator of the slope, $\hat{\beta}_2$ is inconsistent, the estimator for the intercept, $\hat{\beta}_1$, will be too (since $plim\ \hat{\beta}_2 \neq \beta_2$). Saying that would have been enough, but if you want to show it mathematically, first substitute for x:

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2(\bar{z} + \bar{w}) = [\bar{y} - \hat{\beta}_2 \bar{z}] - \hat{\beta}_2 \bar{w}$$

Let the sample go to infinity:

$$plim\ \hat{\beta}_1 = plim\ [\bar{y} - \hat{\beta}_2 \bar{z}] - plim\ \hat{\beta}_2 \bar{w}$$

$$plim\ \hat{\beta}_1 = plim\bar{y} - (plim\hat{\beta}_2)(plim\bar{z}) - plim\ \hat{\beta}_2 \bar{w}$$

$$plim\ \hat{\beta}_1 = plim\bar{y} - (\beta_2 - \beta_2 \frac{\sigma_w^2}{\sigma_x^2})(plim\bar{z}) - plim\ \hat{\beta}_2 \bar{w}$$

$$plim\ \hat{\beta}_1 = (plim\bar{y} - \beta_2 plim\bar{z}) + \beta_2 \frac{\sigma_w^2}{\sigma_x^2}(plim\bar{z}) - plim\ \hat{\beta}_2 \bar{w}$$

$$plim\ \hat{\beta}_1 = \beta_1 + \beta_2 \frac{\sigma_w^2}{\sigma_x^2}(plim\bar{z}) - plim\hat{\beta}_2 \bar{w}$$

$$plim\ \hat{\beta}_1 = \beta_1 + \beta_2 \frac{\sigma_w^2}{\sigma_x^2}\mu_z - (\beta_2 - \beta_2 \frac{\sigma_w^2}{\sigma_x^2})\mu_w$$

$$plim\ \hat{\beta}_1 = \beta_1 - \beta_2[\mu_w + \frac{\sigma_w^2}{\sigma_x^2}(\mu_w + \mu_z)]$$

Thus, β_1 is estimated inconsistently.