Consider the following simple Keynesian macroeconomic model of the U.S. economy.

$$Y_t = C_t + I_t + G_t + NX_t$$

$$C_t = \beta_0 + \beta_1 YD_t + \beta_2 C_{t-1} + \varepsilon_{1t}$$

$$YD_t = Y_t - T_t$$

$$I_t = \beta_3 + \beta_4 Y_t + \beta_5 r_{t-1} + \varepsilon_{2t}$$

$$r_t = \beta_6 + \beta_7 Y_t + \beta_8 M_t + \varepsilon_{3t}$$

where:

Y_t = gross domestic product (GDP) in year t
C_t = total personal consumption in year t
I_t = total gross private domestic investment in year t
G_t = government purchases of goods and services in year t
NX_t = net exports of goods and services (exports - imports) in year t
T_t = taxes in year t
r_t = the interest rate in year t
M_t = the money supply in year t
YD_t = disposable income in year t

The endogenous variables are Y_t, C_t, I_t, YD_t, and r_t. The exogenous and predetermined variables are Gt, NXt, Ct-1, Tt, rt-1, and Mt. Find the reduced form equations for this model.

Answer: Sub the second equation into the first:

$$Y_t = (\beta_0 + \beta_1 YD_t + \beta_2 C_{t-1} + \varepsilon_{1t}) + I_t + G_t + NX_t$$

Now sub for I_t and YD_t :

$$Y_t = (\beta_0 + \beta_1 (Y_t - T_t) + \beta_2 C_{t-1} + \varepsilon_{1t}) + (\beta_3 + \beta_4 Y_t + \beta_5 r_{t-1} + \varepsilon_{2t}) + G_t + NX_t$$

Now solve for Y_t :

$$Y_t = [1/(1\text{-}\beta_1\text{-}\beta_4)]\{(\beta_0 + \beta_3) - \beta_1 T_t + \beta_2 C_{t-1} + \beta_5 r_{t-1} + G_t + NX_t + \varepsilon_{1t} + \varepsilon_{2t}\}$$

And, in the form that would be estimated:

$$Y_t = \lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt}$$

Now substitute this solution into the other equations to find the rest of the reduced form relationships. Consumption first. Sub for disposable income, then income:

$$C_t = \beta_0 + \beta_1(Y_t - T_t) + \beta_2 C_{t-1} + \varepsilon_{1t}$$

$$C_t = \beta_0 + \beta_1[(\lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt}) - T_t] + \beta_2 C_{t-1} + \varepsilon_{1t}$$

$$C_t = (\beta_0 + \beta_1\lambda_0) + \beta_1(\lambda_1 - 1)\, T_t + (\beta_1\lambda_2 + \beta_2) C_{t-1} + \beta_1\lambda_3 r_{t-1} + \beta_1\lambda_4 G_t + \beta_1\lambda_5 NX_t + (\beta_1\varepsilon_{yt} + \varepsilon_{1t})$$

Next, YD_t:

$$YD_t = Y_t - T_t = Y_t = \lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt} - T_t$$

$$YD_t == \lambda_0 + (\lambda_1 - 1) T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt}$$

Investment is next on the list:

$$I_t = \beta_3 + \beta_4 Y_t + \beta_5 r_{t-1} + \varepsilon_{2t}$$

$$I_t = \beta_3 + \beta_4(\lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt}) + \beta_5 r_{t-1} + \varepsilon_{2t}$$

$$I_t = (\beta_3 + \beta_4\lambda_0) + \beta_4\lambda_1 T_t + \beta_4\lambda_2 C_{t-1} + (\beta_4\lambda_3 + \beta_5) r_{t-1} + \beta_4\lambda_4 G_t + \beta_4\lambda_5 NX_t + (\beta_4\varepsilon_{yt} + \varepsilon_{2t})$$

And finally, the interest rate:

$$r_t = (\beta_6 + \beta_7\lambda_0) + \beta_7\lambda_1 T_t + \beta_7\lambda_2 C_{t-1} + \beta_7\lambda_3 r_{t-1} + \beta_7\lambda_4 G_t + \beta_7\lambda_5 NX_t + \beta_8 M_t + (\beta_7\varepsilon_{yt} + \varepsilon_{3t})$$