Consider the following simple Keynesian macroeconomic model of the U.S. economy.

\[ Y_t = C_t + I_t + G_t + NX_t \]

\[ C_t = \beta_0 + \beta_1 YD_t + \beta_2 C_{t-1} + \varepsilon_{1t} \]

\[ YD_t = Y_t - T_t \]

\[ I_t = \beta_3 + \beta_4 Y_t + \beta_5 r_{t-1} + \varepsilon_{2t} \]

\[ r_t = \beta_6 + \beta_7 Y_t + \beta_8 M_t + \varepsilon_{3t} \]

where:

\[ Y_t = \text{gross domestic product (GDP) in year } t \]

\[ C_t = \text{total personal consumption in year } t \]

\[ I_t = \text{total gross private domestic investment in year } t \]

\[ G_t = \text{government purchases of goods and services in year } t \]

\[ NX_t = \text{net exports of goods and services (exports - imports) in year } t \]

\[ T_t = \text{taxes in year } t \]

\[ r_t = \text{the interest rate in year } t \]

\[ M_t = \text{the money supply in year } t \]

\[ YD_t = \text{disposable income in year } t \]

The endogenous variables are \( Y_t, C_t, I_t, YD_t, \) and \( r_t. \) The exogenous and predetermined variables are \( G_t, N X_t, C_{t-1}, T_t, r_{t-1}, \) and \( M_t. \) Find the reduced form equations for this model.

**Answer:** Sub the second equation into the first:

\[ Y_t = (\beta_0 + \beta_1 YD_t + \beta_2 C_{t-1} + \varepsilon_{1t}) + I_t + G_t + NX_t \]

Now sub for \( I_t \) and \( YD_t \):

\[ Y_t = (\beta_0 + \beta_1 (Y_t - T_t) + \beta_2 C_{t-1} + \varepsilon_{1t}) + (\beta_3 + \beta_4 Y_t + \beta_5 r_{t-1} + \varepsilon_{2t}) + G_t + NX_t \]

Now solve for \( Y_t \):

\[ Y_t = \frac{1}{1 - (\beta_1 + \beta_4)} \{(\beta_0 + \beta_3) - \beta_1 T_t + \beta_2 C_{t-1} + \beta_5 r_{t-1} + G_t + NX_t + \varepsilon_{1t} + \varepsilon_{2t} \} \]

And, in the form that would be estimated:
\[ Y_t = \lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt} \]

Now substitute this solution into the other equations to find the rest of the reduced form relationships. Consumption first. Sub for disposable income, then income:

\[ C_t = \beta_0 + \beta_1(Y_t - T_t) + \beta_2 C_{t-1} + \varepsilon_{1t} \]

\[ C_t = \beta_0 + \beta_1[(\lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt}) - T_t] + \beta_2 C_{t-1} + \varepsilon_{1t} \]

\[ C_t = (\beta_0 + \beta_1 \lambda_0) + \beta_1(\lambda_1 - 1) T_t + (\beta_1 \lambda_2 + \beta_2) C_{t-1} + \beta_1 \lambda_3 r_{t-1} + \beta_1 \lambda_4 G_t + \beta_1 \lambda_5 NX_t + (\beta_1 \varepsilon_{yt} + \varepsilon_{1t}) \]

Next, \( YD_t \):

\[ YD_t = Y_t - T_t = Y_t = \lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt} - T_t \]

\[ YD_t = \lambda_0 + (\lambda_1 - 1) T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt} \]

Investment is next on the list:

\[ I_t = \beta_3 + \beta_4 Y_t + \beta_5 r_{t-1} + \varepsilon_{2t} \]

\[ I_t = \beta_3 + \beta_4(\lambda_0 + \lambda_1 T_t + \lambda_2 C_{t-1} + \lambda_3 r_{t-1} + \lambda_4 G_t + \lambda_5 NX_t + \varepsilon_{yt}) + \beta_5 r_{t-1} + \varepsilon_{2t} \]

\[ I_t = (\beta_3 + \beta_4 \lambda_0) + \beta_4 \lambda_1 T_t + \beta_4 \lambda_2 C_{t-1} + (\beta_4 \lambda_3 + \beta_5) r_{t-1} + \beta_4 \lambda_4 G_t + \beta_4 \lambda_5 NX_t + (\beta_4 \varepsilon_{yt} + \varepsilon_{2t}) \]

And finally, the interest rate:

\[ r_t = (\beta_6 + \beta_7 \lambda_0) + \beta_7 \lambda_1 T_t + \beta_7 \lambda_2 C_{t-1} + \beta_7 \lambda_3 r_{t-1} + \beta_7 \lambda_4 G_t + \beta_7 \lambda_5 NX_t + \beta_8 M_t + (\beta_7 \varepsilon_{yt} + \varepsilon_{3t}) \]