Economics 421/521
Winter 2009
Midterm

Answer the following questions (20 points each):

1. Suppose you estimate the following model (standard errors in parentheses):

$$\hat{Y}_i = 4.89 - .87 * X_{2i} - 6.00 * X_{3i} - 11.84 * X_{4i} + u_i$$
$$(1.88)\ (.32) \qquad (2.00) \qquad (6.31)$$
$$N = 20,\ R^2 = .47,\ RSS = 221$$

(a) Form the t-statistic to test the hypothesis that the coefficient on X_3 is negative, i.e. $H_0 : \beta_3 = 0$ versus $H_1 : \beta_3 < 0$. Find the t-statistic in the attached table (use a 1% level of significance), and conduct the test. (b) In the model $Y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + u_i$ describe in detail how to test the hypothesis that $H_0 : 4\beta_2 - 2\beta_3 = 6$ versus $H_1 : 4\beta_2 - 2\beta_3 \neq 6$.

2. (a) What are the consequences of estimating a heteroskedastic model with OLS? (b) Suppose that

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

and that you suspect heteroskedasticity of the form:

$$\sigma_i = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i}$$

Describe how to conduct an LM test for the presence of heteroskedasticity.

3. (a) Suppose that you have tested for heteroskedasticity in the following model, and found it. Describe how to correct the model so that the parameter estimates will be BLUE:

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i, \qquad \sigma_i^2 = \sigma^2 X_{2i}^2$$

(b) Show that the errors in the transformed model, i.e. the errors in the model that is estimated, are homoskedastic.

-- Exam Continues on the Next Page --

4. If we estimate the following models using OLS, what problems, if any, will we encounter? List the problems associated with each model separately:

(a) $Y_t = \alpha Y_{t-1} + \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$

(b) $Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t, \quad u_t = \rho u_{t-1} + e_t$

(c) $Y_t = \alpha Y_{t-1} + \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t, \quad u_t = \rho u_{t-1} + e_t$

(d) The Durbin-Watson statistic is:

$$DW = \frac{\sum_{t=2}^{T} (u_t - u_{t-1})^2}{\sum_{t=1}^{T} (u_t)^2}$$

Show that the Durbin-Watson statistic is between 0 and 4, and that it is 0 with perfect positive correlation, 2 when there is no correlation at all, and 4 when there is perfect negative correlation. Can the Durbin-Watson test be used on model (c)? Why or why not?

5. (a) Give a step by step description of how to conduct a Breusch-Pagan test for higher order serial correlation in the following model:

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t, \quad u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + \rho_4 u_{t-4} + e_t$$

(b) What is autoregressive conditional heteroskedasticity (ARCH)?

Table A.1 (*Continued*)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

Table A.2 t distribution: critical values of t

Degrees of freedom	Two-tailed test: / One-tailed test:	Significance level 10% / 5%	5% / 2.5%	2% / 1%	1% / 0.5%	0.2% / 0.1%	0.1% / 0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725