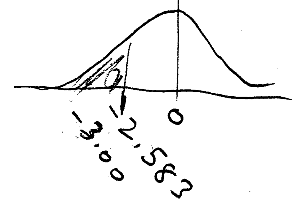
Economics 421
Winter 2009
Solution to Midterm

① (a) $t = \frac{-6-0}{2} = -3.00$



Critical value (1%) $= -2.583$
$\Rightarrow$ Reject null that $\beta_3 = 0$

(b) $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$ (UR)

$$\beta_3 = 2\beta_2 - 3$$

$$y_i = \beta_1 + \beta_2 X_{2i} + (2\beta_2 - 3) X_{3i} + u_i$$
$$= \beta_1 + \beta_2 X_{2i} + 2\beta_2 X_{3i} - 3X_{3i} + u_i$$
$$y_i + 3X_{3i} = \beta_1 + \beta_2 (X_{2i} + 2X_{3i}) + u_i$$
$$y_i^* = \beta_1 + \beta_2 X_{2i}^* + u_i \quad \text{(R)}$$
$$y_i^* = y_i + 3X_{3i}$$
$$X_{2i}^* = X_{2i} + 2X_{3i}$$

To Test:

1. Run UR model above, save RSS_{UR}
2. Run R (Restricted) model above, save RSS_R
3. Form the F-stat (i.e., one restriction) $\dfrac{\dfrac{RSS_R - RSS_{UR}}{1}}{\dfrac{RSS_{UR}}{N-3}}$
4. This is distributed $F(1, N-3)$, so use this critical value to conduct the test.

(2) (a) The coefficient estimates are unbiased and consistent, but the estimates of the standard errors are biased. The biased standard errors also causes the test statistics to be biased

(b) Here are the steps

1. Estimate the model $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$ with OLS, save the residuals, $\hat{u}_i$.
2. Take the absolute value of the $\hat{u}_i$ terms (Std. errors can't be negative), then run the regression
$$|\hat{u}_i| = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \text{error}.$$
3. From the regression output, form the statistic NR^2. This is distributed χ^2(# Restrictions) where, in this case, the number of restrictions is 2 (i.e., the null Hypothesis is $\alpha_2 = \alpha_3 = 0$, the alternative is that one of the two, at least, are non-zero).

(3) $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$, $\sigma_i^2 = \sigma^2 X_{2i}^2$

(a) Steps

1. Divide through by X_{2i}:

$$\frac{Y_i}{X_{2i}} = \beta_1\left(\frac{1}{X_{2i}}\right) + \beta_2 + \beta_3 \frac{X_{3i}}{X_{2i}} + \frac{u_i}{X_{2i}}$$

Renaming the variables:

$$Y_i^* = \beta_1 C_i^* + \beta_2 + \beta_3 X_{3i}^* + u_i^*$$

$$Y_i^* = \frac{Y_i}{X_{2i}}, \quad C_i^* = \frac{1}{X_{2i}},$$

$$X_{3i}^* = \frac{X_{3i}}{X_{2i}}, \quad u_i^* = \frac{u_i}{X_{2i}}$$

This Regression will give BLUE estimates of the parameters.

(b) $Var(u_i^*) = Var\left(\frac{u_i}{X_{2i}}\right) = \frac{1}{X_{2i}^2} Var(u_i)$

$= \frac{1}{X_{2i}^2}(\sigma^2 X_{2i}^2) = \sigma^2$ So,

the errors are homoskedastic.

(4) (a) The estimates from this model are biased, but consistent (Both the coefficient estimates and the std. errors)

(b) The estimates of the coefficients are unbiased and consistent, but the standard errors (and hence the test statistics) are wrong.

(c) The coefficient estimates and standard errors are both biased and inconsistent.

(d) $$DW = \frac{\Sigma (u_t - u_{t-1})^2}{\Sigma u_t^2} = \frac{\Sigma (u_t^2 - 2u_t u_{t-1} + u_{t-1}^2)}{\Sigma u_t^2}$$

$$= \frac{\frac{N-1}{N-1} \Sigma (\cdot)}{\frac{N}{N} \Sigma (\cdot)} = \frac{N-1}{N} \frac{\Sigma \left(\frac{u_t^2}{N-1} - \frac{2u_t u_{t-1}}{N-1} + \frac{u_{t-1}^2}{N-1} \right)}{\Sigma \frac{u_t^2}{N}}$$

as $N \to \infty$, these converge as follows $\left(\frac{N-1}{N} \to 1\right)$

$$DW = \frac{\sigma^2 - 2\,cov(u_t, u_{t-1}) + \sigma^2}{\sigma^2} = 2 - 2\frac{cov}{var}$$

$= 2 - 2\rho$ where ρ is the correlation between u_t and u_{t-1}, when $\rho = 1$ (perfect corr) $DW = 0$, when $\rho = 0$ (no corr) $DW = 2$, and when

[cont.]

(4) [cont.]

$\rho = -1$ (perfect neg. corr), $DW = 4$

(c) No, the DW test cannot be used on model (c) since the $\hat{u}_t$ estimates are biased (due to the correlation between Y_{t-1} and u_{t-1}).

(5) (a) Here are the steps

1. Estimate the model

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$$

using OLS, save the residuals. ($\hat{u}_t$)

2. Run the regression

$$\hat{u}_t = \alpha_1 \hat{u}_{t-1} + \alpha_2 \hat{u}_{t-2} + \alpha_3 \hat{u}_{t-3} + \alpha_4 \hat{u}_{t-4} + \alpha_5 X_{2t} + \alpha_6 X_{3t} + \text{error}.$$

3. From the output, form the statistic $(T-4)R^2$. This is a $\chi^2(4)$.

T → # of observations

(b) ARCH errors are when the variance tends to vary persistently, e.g.

$$\sigma_t^2 = \alpha_0 + \alpha_1 \sigma_{t-1}^2 + \dots + \alpha_p \sigma_{t-p}^2 .$$

So, when the variance evolves autoregressively, this is a useful model.