Economics 421/521
Final Exam
Winter 2011

① (a) To test for heteroskedasticity, first run the regression

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$$

and save the estimated residuals, $\hat{u}_i$. Next, square the estimated residual to obtain $\hat{u}_i^2$, and run the regression $\hat{u}_i^2 = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \text{error}$

The test statistic is NR^2, and if $NR^2 > \chi^2_\alpha(2)$, where α is the predetermined level of significance, then $H_0: \alpha_2 = \alpha_3 = 0$ is rejected (the alternative is $H_1: \alpha_2$ or $\alpha_3 \neq 0$)

(b) To correct the model, regress $\hat{u}_t^2$ on a constant, X_2, and X_3 as above, then use the model to obtain the predicted values:

$$\hat{\sigma}_i^2 = \hat{\alpha}_1 + \hat{\alpha}_2 X_{2i} + \hat{\alpha}_3 X_{3i}$$

(If any of these are negative, use the absolute value). Divide by $\hat{\sigma}_i$:

$$\frac{Y_i}{\hat{\sigma}_i} = \beta_1\left(\frac{1}{\hat{\sigma}_i}\right) + \beta_2\left(\frac{X_{2i}}{\hat{\sigma}_i}\right) + \beta_3\left(\frac{X_{3i}}{\hat{\sigma}_i}\right) + \frac{u_i}{\hat{\sigma}_i}$$ → now homoskedastic

BLUE estimates can be obtained from OLS of this model.

② (a)

$$DW = \frac{\sum_{t=2}^{T} (u_t - u_{t-1})^2}{\sum_{t=1}^{T} (u_t)^2} = \frac{\Sigma (u_t^2 + u_{t-1}^2 - 2u_t u_{t-1})}{\Sigma (u_t)^2}$$

$$= \frac{N-1 \left\{ \frac{1}{N-1} \Sigma (u_t^2 + u_{t-1}^2 - 2u_t u_{t-1}) \right\}}{N \left\{ \frac{1}{N} \Sigma (u_t^2) \right\}}$$

Take plim $\left\{ \text{note that } \operatorname{plim}\left(\frac{N-1}{N}\right) = 1 \right\}$

$$\operatorname{plim} \frac{N-1 \{ \cdot \}}{N \{ \cdot \}} = (1) \frac{\sigma^2 + \sigma^2 - 2\operatorname{cov}(u_t, u_{t-1})}{\sigma^2}$$

$$= 2 - 2\frac{\operatorname{cov}(u_t, u_{t-1})}{\operatorname{var}(u_t)} = 2(1 - \rho)$$

$\rho \hookrightarrow \operatorname{corr}(u_t, u_{t-1})$

① $\rho = 0$ (no corr) $DW = 2$
② $\rho = 1$ (perfect pos) $DW = 0$
③ $\rho = -1$ (perfect neg) $DW = 4$

(b) When $Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 Y_{t-1} + u_t$, $u_t = \rho u_{t-1} + e_t$, then Y_{t-1} and u_t are correlated (since both depend upon u_{t-1}). Since a right-hand side variable, Y_{t-1}, is correlated with the error, u_t, $\rightarrow$ coeff estimate is biased. Since the $\hat{\beta}$'s are biased, $\hat{u}$ is biased $\rightarrow$ biased DW. The solution is to use

$$\text{Durbin's } h = \hat{\rho} \sqrt{\frac{T}{1 - T\sigma_{\hat{\beta}_{Y_{t-1}}}}} \sim N(0,1)$$

$\hat{\rho} = 1 - \frac{1}{2}DW$, $\sigma_{\hat{\beta}_{Y_{t-1}}}$ = std error of coeff. on Y_{t-1}.

(3) (a) $Y_t = \beta_0 + \beta_1 (X_t - W_t) + V_t$

$$Y_t = \beta_0 + \beta_1 X_t + \underbrace{V_t - \beta_1 W_t}_{\text{call this } U_t}$$

$$\hat{\beta}_1 = \beta_1 + \frac{Cov(X, U)}{Var(X)} = \beta_1 + \frac{Cov(X^* + W, V - \beta_1 W)}{Var(X^* + W)}$$

$$\hat{\beta}_1 = \beta_1 + \left(\frac{-\beta_1 \sigma_W^2}{\sigma_{X^*}^2 + \sigma_W^2} \right) = \beta_1 \left(1 - \frac{\sigma_W^2}{\sigma_{X^*}^2 + \sigma_W^2} \right)$$

$$= \left(\frac{\sigma_{X^*}^2}{\sigma_{X^*}^2 + \sigma_W^2} \right) (\beta_1)$$ So, biased.

(b) When the dependent variable is measured with error, it does not cause big problems. The main effect is to add to the overall error in the model (e.g. V_t becomes $V_t + W_t$). This makes estimates noisier, but given the additional error the estimates are still BLUE.

(4) (a) $Y_t = a + bW_t + CA_t + dB_t + u_t$

$W_t = g + hY_t + KF_t + v_t$

$Y_t = a + b\{g + hY_t + KF_t + v_t\} + cA_t + dB_t + u_t$

$\rightarrow Y_t = \frac{1}{1-bh}\left[(a+bg) + bKF_t + cA_t + dB_t + (bv_t + u_t)\right]$

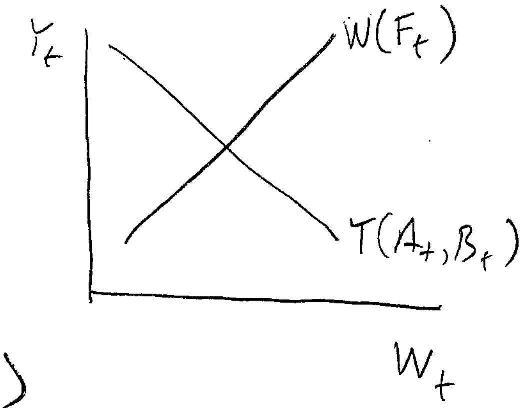
$Y_t = \lambda_0 + \lambda_1 F_t + \lambda_2 A_t + \lambda_3 B_t + \tilde{u}_t$

$W_t = g + h\{\lambda_0 + \lambda_1 F_t + \lambda_2 A_t + \lambda_3 B_t + \tilde{u}_t\} + KF_t + v_t$

$\rightarrow W_t = (g + h\lambda_0) + (h\lambda_1 + k)F_t + h\lambda_2 A_t + h\lambda_3 B_t + (\tilde{u}_t h + v_t)$

$W_t = \delta_0 + \delta_1 F_t + \delta_2 A_t + \lambda_3 B_t + \tilde{V}_t$

(b) {The signs of the slopes of the curves are not specified, so other examples are possible}



(1) When F_t changes, this will map out (ID) the Y-curve (1st eq.). So it is ID.

(2) When A_t changes, W will be mapped out, and another curve will be mapped by B_t. So, the W eq. is over ID → 2 eqs are mapped out (ID). The actual estimates would, essentially, average the two curves

(c) Notice that the second equation, the one for

{cont.}

W_t, has Y_t on the Right-hand side. From the first equation, Y_t has u_t in it. Therefore, W_t is a function of u_t (W_t has u_t in it). But, since W_t has u_t in it, W_t is correlated with the error in the 1st equation, so the estimate of b is biased

$$Y_t = a + bW_t + cA_t + dB_t + u_t$$

↳ this has u_t in it, so W_t is correlated with the error term → bias

The model should be estimated with 2SLS.

(5) (a) The IV should be (1) correlated with the variable it is instrumenting for, (2) uncorrelated with the error term, and (3) not one of the X^{s} already in the model.

(b) Estimation by OLS would lead to biased and inconsistent estimates of the parameters β_1 and β_2. This also biases test statistics.

(c) $$\hat{\beta}_2^{IV} = \frac{\sum_{i=1}^{N}(Y_i-\bar{Y})(Z_i-\bar{Z})}{\sum_{i=1}^{N}(X_i-\bar{X})(Z_i-\bar{Z})} \qquad Z_i = IV$$

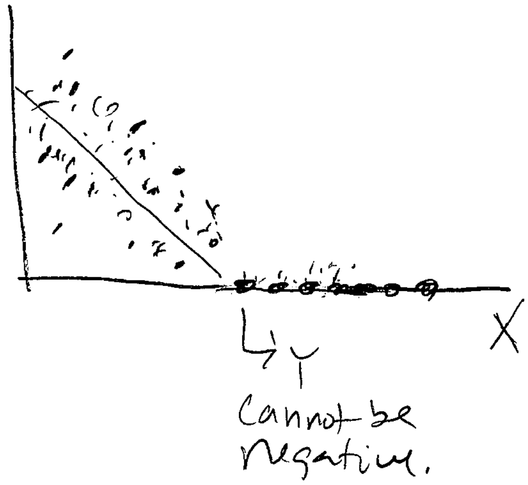
$$\hat{\beta}_2^{IV} = \frac{\beta_2\sum(X_i-\bar{X})(Z_i-\bar{Z})}{\sum(X_i-\bar{X})(Z_i-\bar{Z})} + \frac{\sum(U_i-\bar{u})(Z_i-\bar{Z})}{\sum(X_i-\bar{X})(Z_i-\bar{Z})}$$

$$\hat{\beta}_2^{IV} = \beta_2 + \frac{\frac{1}{N}\sum(U_i)(Z_i-\bar{Z})}{\frac{1}{N}\sum(X_i-\bar{X})(Z_i-\bar{Z})}$$

$$\text{Plim}\,\hat{\beta}_2^{IV} = \beta_2 + \frac{\text{Cov}(Z,u) \to \text{zero from part (a)}}{\text{Cov}(Z,X) \to \text{non-zero from part (a)}}$$

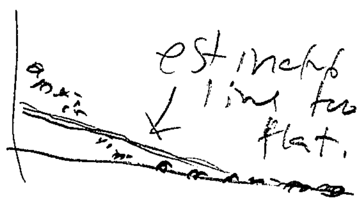
$$= \beta_2$$

⑥ (a) Suppose that the data look as follows Y



(In class, the example was Y = Investment, X = interest rate).

If we fit a line to these data, the slope will be biased, in this case too flat, when OLS is used. Tobit should be used to estimate this model (i.e. MLE).



(b) Maximum Likelihood assumes a set of data are drawn from some distribution, e.g. Normal, with unknown parameters (e.g., for a Normal, the mean and the variance). The estimator finds the params of the distribution (i.e. estimates of mean/variance for normal) most likely to have generated the observed data.

The properties are

① The estimates are consistent (but may be biased in small samples

② The estimates are asymptotically efficient. For Large N, no other estimator has a smaller variance

③ The estimates are asymptotically Normal. (even if underlying dist is non-normal)