Economics 421/521
Midterm Solution
Winter 2011

(a) Unbiased: this says that on average, the estimator is correct. That is, the expected value of the estimator is the true value, $E(\hat{\beta}) = \beta$

(b) Unbiasedness refers to an estimator with a fixed sample size. Consistency asks what happens as the sample size goes to infinity. If the estimator converges to the true value, i.e. $E(\hat{\beta}) \to \beta$, as N goes to infinity, then the estimator is consistent.

(c) Efficient: This means that no other estimator has a smaller variance.

(d) An AR(3) error process is of the form

$$u_t = \rho_1 u_{t-1} + \rho_2 u_{t-2} + \rho_3 u_{t-3} + e_t$$

(e) The definition of the dw statistic is:

$$dw = \frac{\sum_{t=2}^{T} (u_t - u_{t-1})^2}{\sum_{t=1}^{T} (u_t)^2}$$

② (a) The steps are to first estimate the unrestricted model:

$$y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + u_i$$

Next, impose the restriction that $\beta_2 = 1 - \beta_3$ (or that $\beta_3 = 1 - \beta_2$):

$$y_i = \beta_1 + (1 - \beta_3) X_{i2} + \beta_3 X_{i3} + u_i$$

$$y_i = \beta_1 + X_{i2} + \beta_3 (X_{i3} - X_{i2}) + u_i$$

$$y_i - X_i = \beta_1 + \beta_3 (X_{i3} - X_{i2}) + u_i$$

$$y_i^* = \beta_1 + \beta_3 X_i^* + u_i \rightarrow \text{Run this after creating new variables}$$

$$y_i^* = y_i - X_i$$
$$X_i^* = X_{i3} - X_{i2}$$

Finally, form

$$F = \frac{\dfrac{RSS^R - RSS^{UR}}{1}}{\dfrac{RSS^{UR}}{N-K}} \rightarrow \text{Compare to critical value } F_\alpha(1, N-K)$$

1 = # of Rest., K = # of indep. variables

N = Sample size

$H_0: \beta_2 + \beta_3 = 1$

$H_1: \beta_2 + \beta_3 \neq 1$

② (b) The test statistic is NR^2:

$$NR^2 = (40)(.25) = 10$$

The critical value for $\chi^2(5)$
is 11.70

↳ the number of X terms in auxillary Reg.

Ho: all coeff on X terms in the aux regression are zero.

i.e., $\hat{u}_t^2 = \alpha_0 + \alpha_1 X_{i2} + \alpha_2 X_{i3} + \alpha_3 X_{i2}^2 + \alpha_4 X_{i3}^2 + \alpha_5 X_{i2} X_{i3}$.

$H_0: \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$

H_1: at least one non-zero.

Since $10 < 11.70$, fail to Reject.

(c) Two of
① Learning over time → Variance
② inclusion of scale variables like income or sales
③ Better data collection as technology improves
④ outliers
⑤ Model incorrectly specified, e.g. omitted variables

incorrect data transformations, or using a linear model to estimate a non-linear relationship.

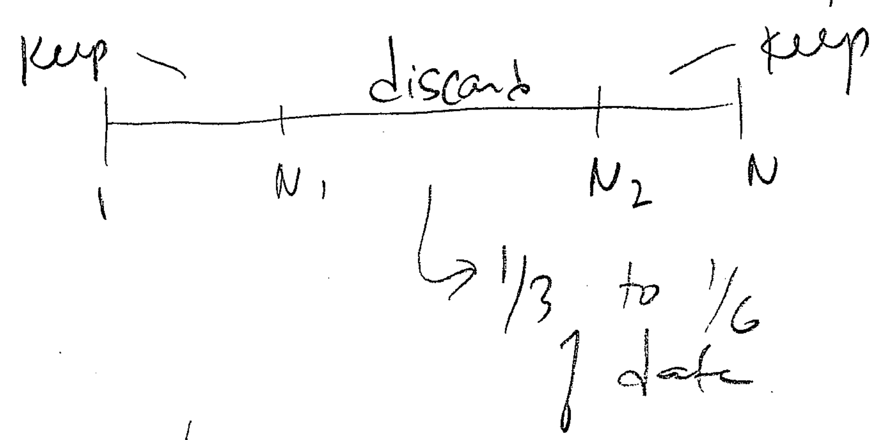
3 (a) The estimates are still unbiased and consistent, but not efficient

(b) The steps are:

1. Identify a variable, say z_i, to which σ_i^2 is suspected of being positively related to, e.g. income.

2 Arrange data from the smallest to largest value of Z_i (i.e. sort data by z_i)

2. Divide the sample into three parts



3. Estimate two regressions, one for the first N_1 observations, and one for the last $N - N_2 + 1$ observations (usually, the sample sizes are equal).

4. $RSS_1 = \sum_{i=1}^{N_1} \hat{u}_i^2$ from $1^{\underline{ST}}$ regression

$RSS_2 = \sum_{i=N-N_2+1}^{N} \hat{u}_i^2$ from $2^{\underline{ND}}$ regression

5. Compute $F = \dfrac{\hat{\sigma}_2^2}{\hat{\sigma}_1^2} = \dfrac{\frac{RSS_1}{N_1 - K}}{\frac{RSS_2}{N-N_2+1-K}}$

$$F \sim F(\underbrace{N-N_2+1-K,\ N_1-K}_{\text{degrees of freedom}})$$

Compare F to critical value from table. $H_0: RSS_1 = RSS_2$ (i.e. variances are equal) $\sigma_1^2 = \sigma_2^2$

H_1 They are not equal.

If $F < F_{critical}$, fail to Reject

If $F > F_{critical}$, Reject.

(4) (a) $Y_i = \beta_1 + \beta_2 X_{i2} + \beta_3 X_{i3} + u_i$

$var(u_i) = \sigma_i^2$.

The correction is to divide by the standard error.

$$\frac{Y_i}{\sigma_i} = \beta_1\left(\frac{1}{\sigma_i}\right) + \beta_2\left(\frac{X_{i2}}{\sigma_i}\right) + \beta_3\left(\frac{X_{i3}}{\sigma_i}\right) + \frac{u_i}{\sigma_i}$$

$$Y_i^* = \beta_1 X_{i1}^* + \beta_2 X_{i2}^* + \beta_3 X_{i3}^* + u_i^*$$

The variance of the transformed model is a constant:

$$var(u_i^*) = E\left(\frac{u_i}{\sigma_i}\right)^2 = \frac{1}{\sigma_i^2} E(u_i)^2$$

$$= \frac{\sigma_i^2}{\sigma_i^2} = 1$$

(b) There are N variances and K parameters

$\rightarrow$ N+K things to estimate. But, cannot estimate N+K things with only N observations

[cont.]

(4) (b) (cont.)

What to do? Reduce the number of variables to estimate by parameterizing model as in part (c).

(c) The three models are:

$\sigma_i^2 = \alpha_1 + \alpha_2 Z_{2i} + \cdots + \alpha_p Z_{pi}$ variance in linear (↑ Linearly)

$\sigma_i = \alpha_1 + \alpha_2 Z_{2i} + \cdots + \alpha_p Z_{pi}$ → Variance ↑ Quadratically in Z's

$\ln \sigma_i^2 = \alpha_1 + \alpha_2 Z_{2i} + \cdots + \alpha_p Z_{pi}$

↳ Variance grows or decays exponentially.

(5) (a) write the model one period ahead

$$y_{t+1} = \beta_1 + \beta_2 x_{t+1,2} + \beta_3 y_t + u_{t+1}$$

↳ this has u_t in it

Thus, one of the r.H.S. variables is correlated with u_t (u_t is correlated with a variable at $t+1$ So the correlation is not contemporaneous).

In this case, the estimator is biased, but consistent and asymptotically efficient.

(b) When the errors are correlated contemporaneously, as they are when $u_t = \rho u_{t-1} + e_t$ (because u_t has u_{t-1} in it, and Y_{t-1} also has u_{t-1} in it, they are correlated at t) the estimator is biased and inconsistent. It is also inefficient.

(c) $y_t = \beta_1 + \beta_2 x_{t2} + \beta_3 x_{t3} + u_t$

$$\rho y_{t-1} = \rho \beta_1 + \rho \beta_2 x_{t-1,2} + \rho \beta_3 x_{t-1,3} + u_{t-1}$$

[cont.)

⑤ (c) (cont)

Subtract the second model from the first

$$y_t - \rho y_{t-1} = B_1 - \rho B_1 + B_2 X_{t2} - \rho B_2 X_{2,t-1} + B_3 X_{t3} - \rho B_3 X_{t-1,3} + u_t - \rho u_{t-1}$$

Key is that this is e_t which is not serially correlated

Rearrange terms

$$y_t = \rho y_{t-1} + B_1(1-\rho) + B_2 X_{t2} - \rho B_2 X_{2,t-1} + B_3 X_{t3} - \rho B_3 X_{3,t-1} + e_t$$

$\text{var } e_t = \sigma^2$ so homoskedastic

This is non-linear in the parameters, so it must be estimated by non-linear least squares

(6) (a) The steps are

1. Estimate the model by OLS

$$y_t = \beta_1 + \beta_2 X_{t2} + \beta_3 X_{t3} + u_t$$

Save $\hat{u}_t$, $t = 1, 2, \ldots, T$

2. Regress $\hat{u}_t$ on X_{t2} and X_{t3} along with $\hat{u}_{t-1} \ldots \hat{u}_{t-p}$

The number of observations is now $T-p$ since the lags cause us to lose the first p obs.

3. Compute $(T-p)R^2 \sim \chi^2(p)$
d.f.

$$H_0: \rho_1 = \rho_2 = \cdots = \rho_p = 0$$

H_1 at least one non-zero

Compare $(T-p)R^2$ to critical value. If > critical reject. If < critical, fail to reject.

(b) The Durbin-Watson test is based upon an assumption that the errors follow a 1st order process:

$$u_t = \rho u_{t-1} + e_t$$

If the errors follow a higher order process that omits u_{t-1}, e.g. a seasonal adjustment such as $u_t = \rho u_{t-4} + e_t$ for quarterly data, the DW test will not pick this up, but the LM test will.

(c) One reason is misspecification, e.g. omitting a serially correlated variable from the RHS (so it's in the error). Another is partial/incomplete/partial adjustment to shocks. A third reason (they only need one) is fitting a linear model to a non-lin relationship

errors +, −, + →
looks like Autocorr.