Economics 421
Winter 2012
Midterm Solution

① (a) $t = \frac{7-1}{3} = 2.00$, critical $t = 2.160$

$df = 17-4 = 13$
Sig. Level = 5%

Since $t < t_{critical}$, fail to reject $H_0: \beta_3 = 1$.

(b) First, estimate the unrestricted (UR) model:

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \beta_4 X_{4i} + u_i$$

Save RSS^{UR} (sum of squared errors)

Next, sub in the restrictions $\beta_2 = \beta_3 = 2.0$

$$Y_i = \beta_1 + 2X_{i2} + 2X_{i3} + \beta_4 X_{i4} + u_i$$

Rearrange:

$$\underbrace{Y_i - 2X_{i2} - 2X_{i3}}_{\tilde{Y}_i} = \beta_1 + \beta_4 X_{i4} + u_i$$

Then, regress $\tilde{Y}_i$ on const, X_4, Save RSS^R (restricted error sum of squares).

Finally Form $F = \dfrac{\frac{RSS^R - RSS^{UR}}{2}}{\frac{RSS^{UR}}{N-4}}$

This is distributed $F(2, N-4)$, and the test is performed as usual (reject if $F > F_{critical}$).

(2) (a) The coefficient estimates are unbiased and consistent, but estimates of the standard errors are biased. The biased std errors also causes biased test statistics.

(b) 1. Estimate the model $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$ by OLS, save the residuals $\hat{u}_i$.

2. Form $\hat{u}_i^2$, and run the regression

$$\hat{u}_i^2 = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \text{error}$$

3. Form the statistic $LM = NR^2$. This is distributed χ^2 (# restrictions) where, in this case, the number of restrictions is 2 (for $H_0: \alpha_2 = \alpha_3 = 0$ vs. H_1: one not zero).

4. Suppose that $NR^2 >$ Critical χ^2, i.e. null rejected. To correct the problem, go back to the auxilliary regression is step 2 and obtain the forecasted value of the dependent variable, call it $\hat{\sigma}_i^2$ (i.e. $\hat{\sigma}_i^2 = \hat{\alpha}_1 + \hat{\alpha}_2 X_{2i} + \hat{\alpha}_3 X_{3i}$ or $\hat{\sigma}_i^2 = \hat{u}^2 - \widehat{\text{error}}$).

5. Take the square root of the forecasted values to get an estimate of σ, call it $\hat{\sigma}$. Then, divide through by $\hat{\sigma}$ (use abs. value if forecast is negative)

(cont.)

(2) [cont.] to get:

$$\frac{Y_i}{\hat{\sigma}_i} = \frac{1}{\hat{\sigma}_i} \beta_1 + \beta_2 \frac{X_{2i}}{\hat{\sigma}_i} + \beta_3 \frac{X_{3i}}{\hat{\sigma}_i} + \frac{u_i}{\hat{\sigma}_i}$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$Y_i^* \qquad X_1^* \qquad X_{2i}^* \qquad X_{3i}^*$$

6. Finally, regress Y_i^* on X_1^*, X_2^*, X_3^* (no constant). These estimates are BLUE.

(3) (a) Bias is a "small sample" or fixed N (fixed sample size) property. It refers to what would happen if you averaged the outcome of repeated experiments all with a sample of size N (flip a coin 10 times, do this over and over, average the results).

Consistency is a large sample property. It's what happens when the sample goes to infinity ($N \to \infty$).

(3) (b) The error term, $u_t = \rho u_{t-1} + e_t$, is correlated contemporaneously with one of the right-hand side variables, so the estimates are biased and inconsistent. The std errors/test statistics are also biased (due to the bias from the correlation between u_t and the RHS and the correlated errors).

(c) Since the estimates of the errors, $\hat{u}_t$, are biased/inconsistent, the DW statistic is also biased/inconsistent. The use of Durbin's - h overcomes this problem.

(4) Given the model

$$Y_i = \beta_1 + \beta_2 X_{2i} + \cdots + \beta_K X_{Ki} + u_i$$

The steps are

1. Estimate the model by OLS, save the residuals $\hat{u}_i$.

[cont.]

4 (a) [cont.]

2. Regress $\hat{u}_t^2$ on a constant, all the X^s (X_{2i} --- X_{Ki}), all the X^s squared (X_{2i}^2 --- X_{Ki}^2), and all of the cross-products ($X_i X_j$, $i, j = 2, ---, K$ and $i < j$)

3. Compute NR^2 (N = sample size). This is distributed as χ^2(# restrictions), where the # rest = # of terms on the right-hand side of the regression in step 2, not counting the constant. For example, for K = 3,

$$\hat{u}_i^2 = \alpha_1 + \alpha_2 X_{2i} + \alpha_3 X_{3i} + \alpha_4 X_{2i}^2 + \alpha_5 X_{3i}^2 + \alpha_6 X_{2i} X_{3i} + \text{error}.$$

rest = 5.

Perform test in usual way. That is, reject $H_0: \alpha_2 = \alpha_3 = \cdots = \alpha_p = 0$ vs. H_1: one not zero if $NR^2 > \chi^2_{\text{critical}}(\#\text{Rest})$

(9) (b) The Steps are

1. Estimate the model by OLS, i.e. estimate

$$Y_t = \beta_1 + \beta_2 X_{2t} + \cdots + \beta_K X_{Kt} + u_t$$

and save $\hat{u}_t$, $t = 1, 2, \cdots, T$, (the estimated residuals).

2. Regress $\hat{u}_t$ on a constant, $X_{2t} \cdots X_{Kt}$, and $\hat{u}_{t-1} \cdots \hat{u}_{t-p}$. Note that the number of observations is now $T-p$ since we lose observations due to the presence of lagged $\hat{u}_t$'s.

3. Compute $(T-p)R^2 \sim \chi^2(\underbrace{p}_{\text{d.f.}})$

for $H_0: \rho_1 = \rho_2 = \cdots = \rho_p = 0$

H_1: at least one non-zero

Compare $(T-p)R^2$ to the critical value for the test. If $(T-p)R^2 >$ critical value, reject. If $<$ critical value, fail to reject.

(5) $Y_i = \beta_1 + \beta_2 V_i + U_i$, $X_i = V_i + e_i$

sub in to get

$$Y_i = \beta_1 + \beta_2 (X_i - e_i) + U_i$$

Rearrange terms

$$Y_i = \beta_1 + \beta_2 X_i + \underbrace{(U_i - \beta_2 e_i)}_{\text{call this } W_i}$$

zero mean

We know that

$$\hat{\beta}_2 = \beta_2 + \frac{\frac{1}{N} \Sigma (X_i - \bar{x}) W_i}{\frac{1}{N} \Sigma (X_i - \bar{x})^2}$$

Then, letting $N \to \infty$,

$$\text{plim } \hat{\beta}_2 = \beta_2 + \frac{\text{Cov}(X, W)}{\text{Var}(X)}$$

$$\text{Cov}(X, U) = \text{Cov}(V+e, u - \beta_2 e) = -\beta_2 \sigma_e^2$$
$$\text{Var}(X) = \text{Var}(V+e) = \sigma_v^2 + \sigma_e^2$$

$\text{Plim } \hat{\beta}_2 = \beta_2 - \beta_2 \left(\frac{\sigma_e^2}{\sigma_v^2 + \sigma_e^2} \right)$. Thus, it is biased and inconsistent.